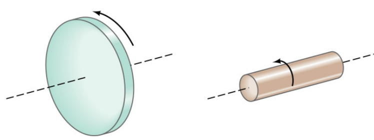
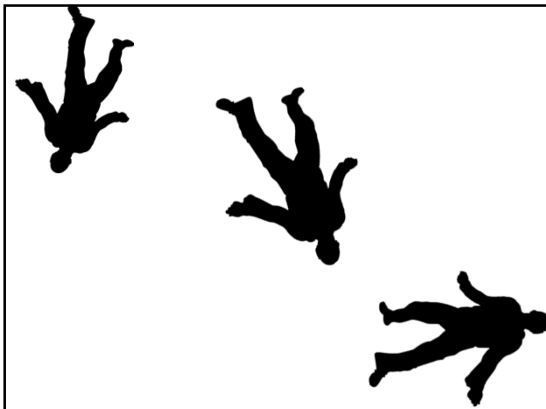
## PA Multi-Region STEM Partnership
## August 2013 Follow-Up
## Rotational Motion



Immaculata University

August 15 & 16, 2013







## What we will cover:

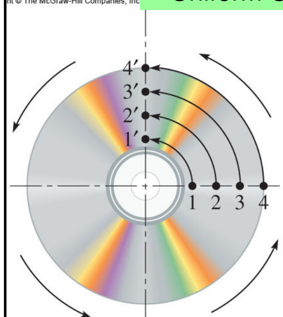
- Circular Motion
- Rotational Inertia
- Torque
- Center of Mass and Center of Gravity
- Centripetal Force

- Later this Year:
  - Angular Momentum
  - Conservation of Angular Momentum





## Uniform Circular Motion



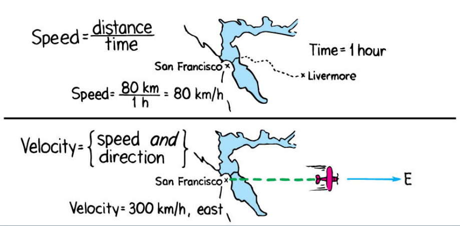

Notice that as the disk rotates, the distance each dot travels depends on where it is on the disk.

1. Rotational speed of CD
2. Tangential speed of each point



Figure 3.11

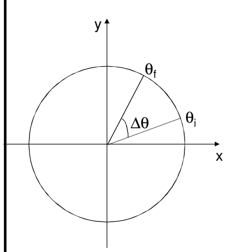
## Linear Motion




Distance : X or d

Velocity: v

## Circular Motion-Terminology




θ is the angular position.

Angular displacement:

$$\Delta\theta = \theta_f - \theta_i$$

θ is measured in radians.

2π radians = 360° = 1 revolution

Angles measured *Clockwise* are negative (-) and angles measured *Counter Clockwise* are positive (+).

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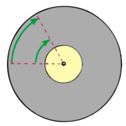
## Circular Motion – Rotational Speed

- Rotational (angular) speed is the *number of rotations or revolutions per unit of time*
- **symbol *ω***
- ***ω = ΔΘ/Δt***



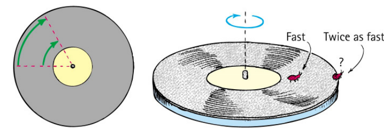
(a)

- All parts of a rigid merry-go-round or turntable turn about the axis of rotation in the same amount of time.
- So, all parts have the same rotational speed.



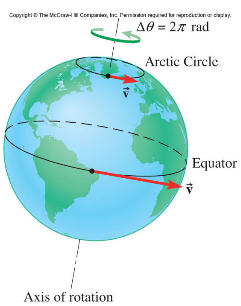
## Circular Motion—Tangential Speed

- The distance traveled by a point on the rotating object divided by the time taken to travel that distance is called its *tangential* speed (symbol *v*).
- Points closer to the circumference have a greater tangential speed than points closer to the center.




(a) (b)






A person at the Arctic Circle is *rotating* as fast as a person at the Equator. Their *angular speed ω* is the same, *HOWEVER*, the person at the Equator is moving a lot faster. *Their linear or tangential speed is faster*

$$v = r\omega$$

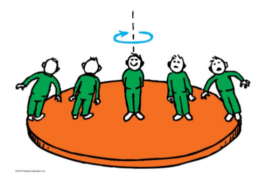
ω is measured in rads/sec



## Relationship between Rotational & Tangential Speed

Tangential speed = Radial Distance × Rotational Speed

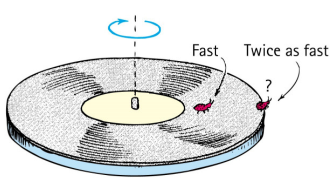
$$v = r\omega$$



Compare the tangential speed of a person at 1 meter from the axis of rotation with a person at 4 meters. The rotational speed ω = 2π rad per second.



## Velocity and Acceleration for Circular Motion



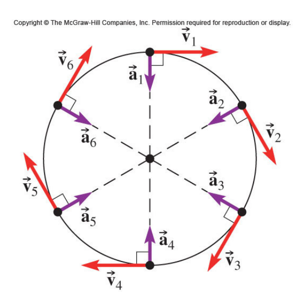

(b)

Notice the direction of each ladybug is continually changing. Therefore it's velocity is changing

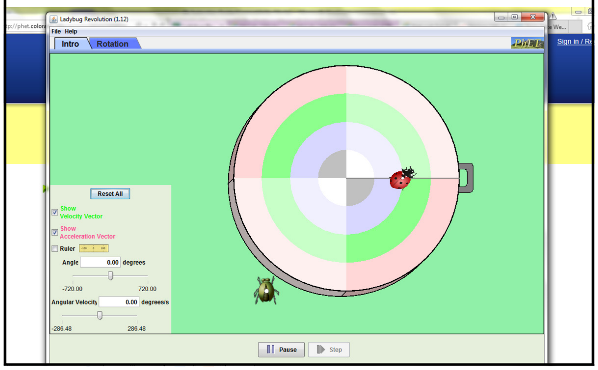


## Centripetal Acceleration changes the Direction

**$a = v^2/r = \omega^2 r$**

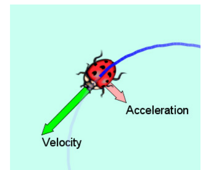


# Ladybug Revolution




# Ladybug Motion

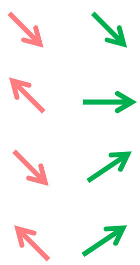
Quick Quiz

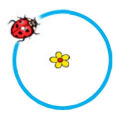





2. What could the **acceleration** and **velocity** vectors look like?

A.
B.
C.
D.

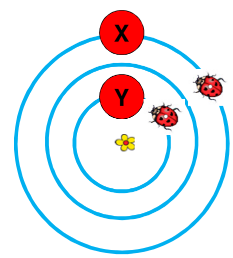






If you had two bugs moving in circles like this, what could the **velocity** vectors at point X vs point Y look like?

| | X | Y |
|---|---|---|
| **A** | ← | ← |
| **B** | ← | ← |
| **C** | ← | ← |
| D | Any of the above | |
| E | None of the above are possible | |

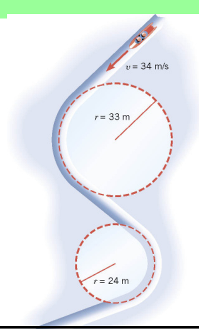





**The Effect of Radius on Centripetal Acceleration**

The bobsled track contains turns with radii of 33 m and 24 m. Find the centripetal acceleration at each turn for a speed of 34 m/s.

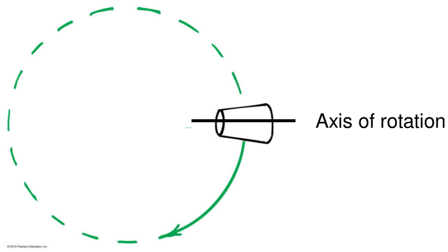
$$a_r = v^2/r$$






Where is the axis of rotation?

Figure 8.3




Inner edge of cup moves slower than outer edge
Outer edge of cup covers more distance per rotation



Figure 8.3

Connect your cups like this:



Figure 8.5

Roll them down your 2 meters sticks.
Discuss what happens and why

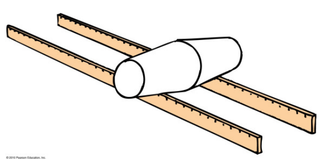




Figure 8.6

Railroad wheels are tapered so they can stay on track

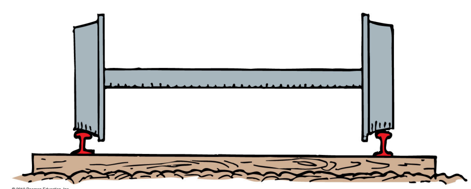




Figure 8.7

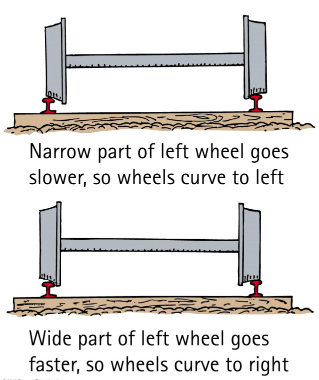
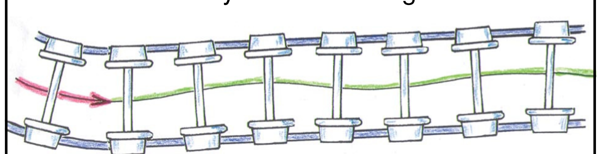





Figure 8.8

Wheels stay on track during turns



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The narrow rim of the wheel moves slower because it is closer to the axis of rotation.
The wider rim of the wheel moves faster because it is further from the axis of rotation
This combined motion turns the train.

## Newton's First Law (The Law of Inertia)

If no force acts on an object, then its speed and direction of motion do not change.


•If the object is at rest, it remains at rest If the object is in motion, it continues to move in a straight line with the same speed.



## Inertia

Here is a lot of inertia. The large train resists changing its motion

Here is very little inertia. The small baby carriage has very little resistance to change in motion.

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## Rotational Inertia

- An object rotating about an axis tends to remain rotating about the same axis at the same rotational speed unless interfered with by some external force.
- The property of an object to resist changes in its rotational state of motion is called **rotational inertia or moment of inertia**

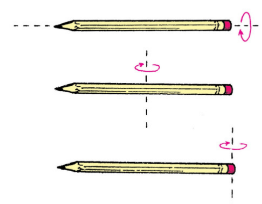


## Try This

Rotate the Same Pencil or Pen about these three axis of rotation

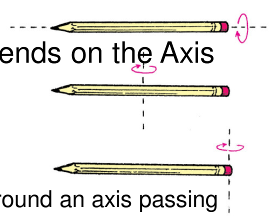
Through which axis is it the easiest to rotate?

Through which axis is it the most difficult to rotate?





## Ease of Rotation depends on the Axis

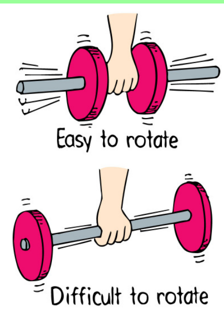


- Easier to rotate pencil around an axis passing through it.
- Harder to rotate it around vertical axis passing through center.
- Hardest to rotate it around vertical axis passing through the end.



## Rotational Inertia

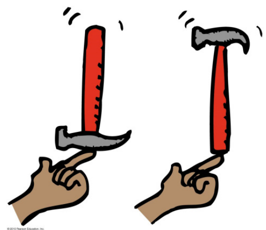
Depends upon
- mass of object.
- distribution of mass around axis of rotation.
  – The greater the distance between an object's mass concentration and the axis, the greater the rotational inertia.



# Try This

Which way is it the easiest to balance the hammer?

Why?

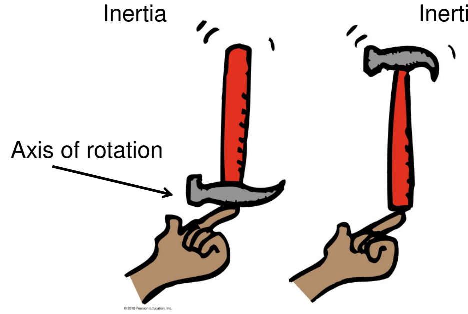




## Same Mass

Smaller Moment of Inertia

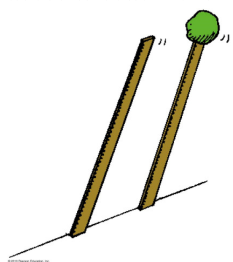
Greater Moment of Inertia








Which stick has the greater rotational inertia about its bottom end?

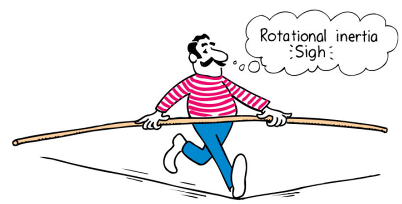


Which stick will rotate to the floor first?



# Rotational Inertia

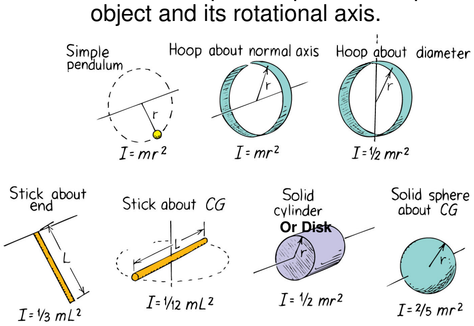
- The greater the rotational inertia, the harder it is to change its rotational state.
  - A tightrope walker carries a long pole that has a high rotational inertia, so it does not easily rotate.
  - Keeps the tightrope walker stable.






# Rotational Inertia I

The rotational inertia depends upon the shape of the object and its rotational axis.



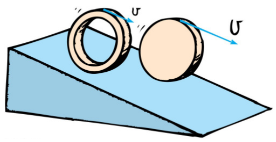



## Comparing Pendulums with different Moments of Inertia

$I = mr^2$

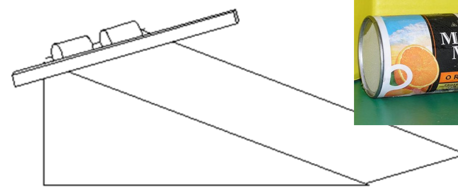
A hoop and a disk are released from the top of an incline at the same time. Which one will reach the bottom first?

A. Hoop
B. Disk
C. Both together
D. Not enough information



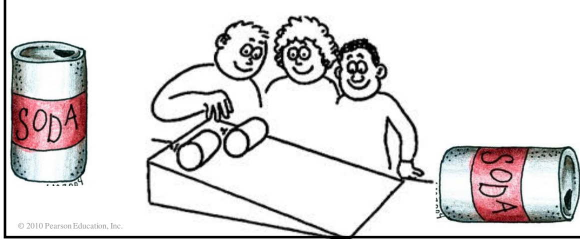


Which will roll down a hill faster, a can of regular fruit juice or a can of frozen fruit juice? Why?






Roll a pair of identical cans of carbonated beverage down an incline. You will find they roll at the same rate. Now shake one of them so bubbles form inside. Repeat the experiment. What do you observe and why?






## *What is Kinetic Energy?*

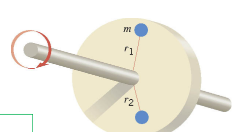
- Energy of Motion:
  - KE = ½ $mv^2$



## Rotational Kinetic Energy

The rotational kinetic energy of a rigid rotating object is

$$KE_R = \tfrac{1}{2} I\omega^2$$




***Requirement:*** The angular speed must be expressed in rad/s.

***SI Unit of Rotational Kinetic Energy:*** joule (J)



## Rolling Objects

An object that is rolling combines translational motion (its center of mass moves) and rotational motion (points in the body rotate around the center of mass).

For a rolling object:

$$KE_{tot} = KE_T + KE_{rot}$$
$$= \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I\omega^2$$

If the object rolls without slipping then $v_{cm} = R\omega$.

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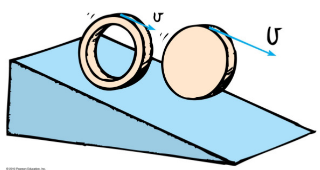
Total Kinetic Energy = Translational Kinetic Energy + Rotational Kinetic Energy

$$KE = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$





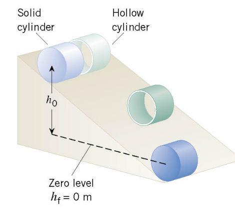
$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 + mgh$$

**ENERGY CONSERVATION**

$$\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2 + mgh_f = \tfrac{1}{2}mv_i^2 + \tfrac{1}{2}I\omega_i^2 + mgh_i$$

$$\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2 = mgh_i$$

$$\omega_f = v_f / r$$

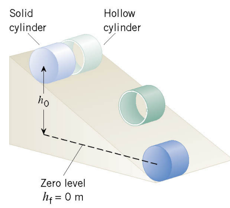





*9.5 Rotational Work and Energy*

$$\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\, v_f^2/r^2 = mgh_i$$

$$v_f = \sqrt{\frac{2mgh_o}{m + I/r^2}}$$

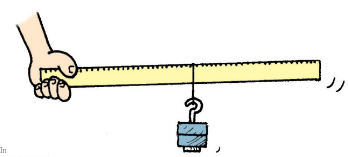



The cylinder with the smaller moment of inertia will use less rotational KE. Therefore there is a greater translational KE and hence, greater final translational speed.



# Torque

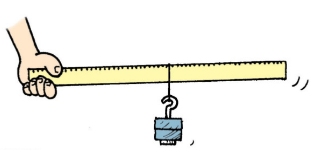
- The tendency of a force to cause rotation is called **torque.**
- Torque depends upon three factors:
  - Magnitude of the force (weight)
  - The direction in which it acts (down)
  - The point at which it is applied on the object





# Torque

- The equation for Torque is
  Torque = lever arm × force
- The lever arm depends upon
  - where the force is applied.
  - the direction in which it acts.



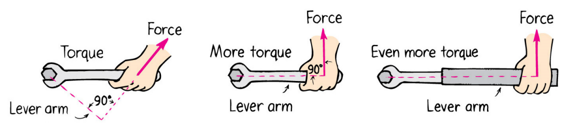


# Lever Arm

- Lever arm is the perpendicular distance from the axis of rotation to the line along which the force acts.

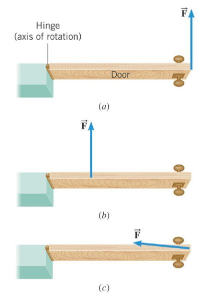
# Lever Arm

- 1st picture: Lever arm is *less than* length of handle because of direction of force.
- 2nd picture: Lever arm is equal to length of handle.
- 3rd picture: Lever arm is longer than length of handle.






## Torque




*Line of Action:* an extended line drawn colinear with the force

*Lever Arm:* perpendicular distance between line of action and axis of rotation

The amount of torque depends on where and in what direction the force is applied, as well as the location of the axis of rotation.



Saw the broom at the center of mass. What weights more, the broom handle or the brushes? Or do they weigh the same? Why?

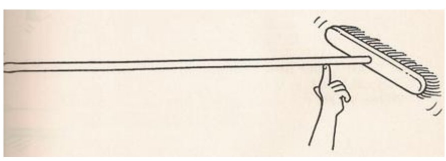
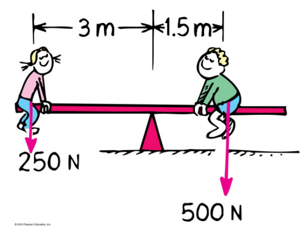




Figure 8.18

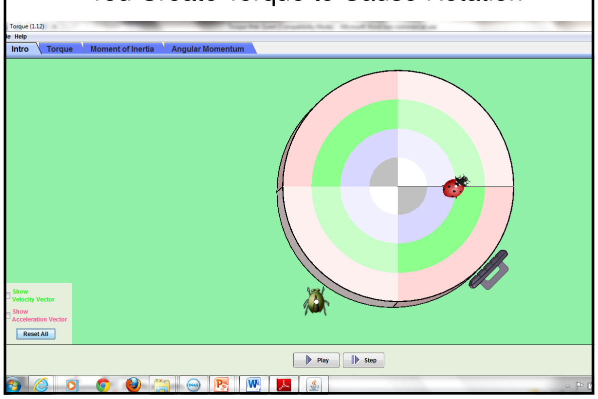
## Torque Causes Rotation




Equal and opposite torque results in equilibrium



## You Create Torque to Cause Rotation



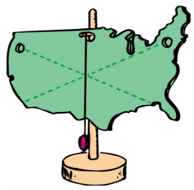

## Center of Mass and Center of Gravity

- **Center of mass** is the average position of all the mass that makes up the object.
- **Center of gravity (CG)** is the average position of weight distribution.
  - Since weight and mass are proportional, center of gravity and center of mass usually refer to the same point of an object.

## Center of Mass and Center of Gravity

To determine the center of gravity,
- suspend the object from a point and draw a vertical line from suspension point.
- repeat after suspending from another point.

- The center of gravity lies where the two lines intersect.

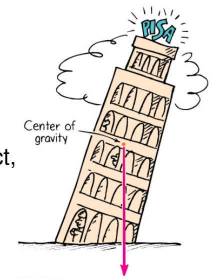




## Center of Gravity—Stability

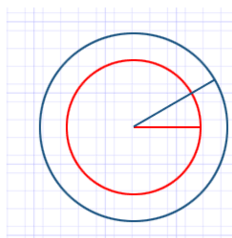
The location of the center of gravity is important for stability.

- If we draw a line straight down from the center of gravity and it falls inside the base of the object, it is in stable **equilibrium;** it will balance.
- If it falls outside the base, it is unstable.






## Centripetal Force: Move the Ping Pong Ball in a Circle





## Centripetal Force

- Any force directed toward a fixed center is called a **centripetal force.**
- *Centripetal* means “center-seeking” or “toward the center.”

Example: To whirl a tin can at the end of a string, you pull the string toward the center and exert a centripetal force to keep the can moving in a circle.



## Centripetal Force

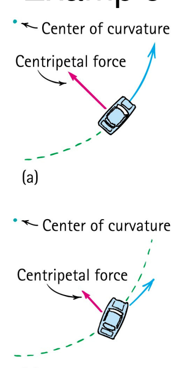
- Depends upon
  - mass of object.
  - tangential speed of the object.
  - radius of the circle.
- In equation form:

$$\text{Centripetal force} = \frac{\text{mass} \times \text{tangential speed}^2}{\text{radius}}$$



## Centripetal Force—Example

- When a car rounds a curve, the centripetal force prevents it from skidding off the road.
- If the road is wet, or if the car is going too fast, the centripetal force is insufficient to prevent skidding off the road.



## Centripetal Force
### CHECK YOUR NEIGHBOR

Suppose you double the speed at which you round a bend in the curve, by what factor must the centripetal force change to prevent you from skidding?

A. Double
B. Four times
C. Half
D. One-quarter



## Centripetal Force
### CHECK YOUR NEIGHBOR

Suppose you take a sharper turn than before and *halve* the radius, by what factor will the centripetal force need to change to prevent skidding?

A. Double
B. Four times
C. Half
D. One-quarter



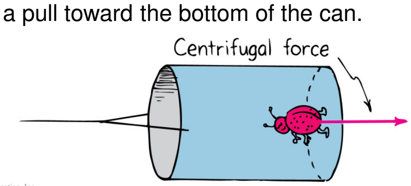
# Centrifugal Force

- Although centripetal force is center directed, an occupant inside a rotating system seems to experience an outward force. This apparent outward force is called **centrifugal force.**
- *Centrifugal* means "center-fleeing" or "away from the center."



# Rotating Reference Frames

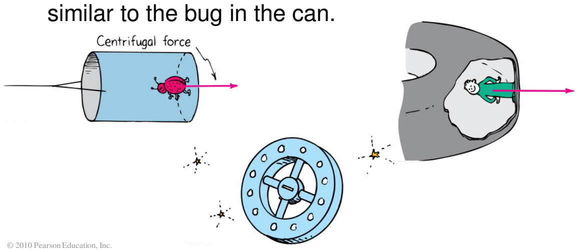
- Centrifugal force *in a rotating reference frame* is a force in its own right – as real as any other force, e.g. gravity.
- Example:
  - The bug at the bottom of the can experiences a pull toward the bottom of the can.






# Simulated Gravity

- Centrifugal force can be used to simulate gravity in space stations of the future.
- By spinning the space station, occupants would experience a centrifugal force (simulated gravity) similar to the bug in the can.

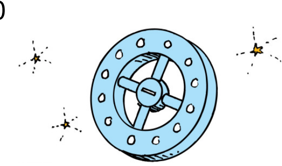





# Simulated Gravity

To simulate an acceleration due to gravity, *g*, which is 10 $m/s^2$, a space station must

- have a radius of about 1 km (i.e. diameter of 2 km).
- rotate at a speed of about 1 revolution per minute.



## Rotational and Tangential Speed
### CHECK YOUR NEIGHBOR

A ladybug sits halfway between the rotational axis and the outer edge of the turntable . When the turntable has a rotational speed of 20 RPM and the bug has a tangential speed of 2 cm/s, what will be the rotational and tangential speeds of her friend who sits at the outer edge?

A. 1 cm/s
B. 2 cm/s
C. 4 cm/s
D. 8 cm/s

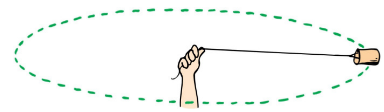
$$v = r\omega$$



## Angular Momentum
### CHECK YOUR NEIGHBOR

Suppose you are swirling a can around and suddenly decide to pull the rope in *half*way; by what factor would the speed of the can change?

A. Double
B. Four times
C. Half
D. One-quarter

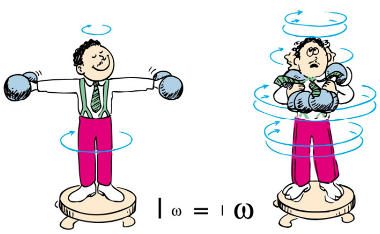




## Angular Momentum
### CHECK YOUR NEIGHBOR

Suppose by pulling the weights inward, the rotational inertia of the man reduces to half its value. By what factor would his angular velocity change?

A. Double
B. Three times
C. Half
D. One-quarter



$$I\omega = I\omega$$



## Rotational Inertia
### CHECK YOUR NEIGHBOR

Suppose the girl on the left suddenly is handed a bag of apples weighing 50 N. Where should she sit order to balance, assuming the boy does not move?

A. 1 m from pivot
B. 1.5 m from pivot
C. 2 m from pivot
D. 2.5 m from pivot