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Exploring Unknown Probabilities with Miniature Toy Pigs

The typical probability experience for middle-grades students involves flipping coins and rolling dice. In both of these situations, probability can be discussed from experimental and theoretical viewpoints. It is not likely that your students have ever experienced a situation in which the theoretical probability is unknown. The following activities investigate the tossing of a toy pig (see **fig. 1**), with an unknown probability of each landing position (Whyte 2006). In other words, the theoretical probability of each landing position cannot be determined. Hence, students contend only with finding an experimental probability for each possible outcome.

The activities described here address the following NCTM Standards (2000, p. 248):

- Develop and evaluate inferences and predictions that are based on data.
- Understand and apply basic concepts of probability.

In this article, we present three activities to promote thinking about probability in middle-grades students.

In **activity 1**, students determine the possible landing positions and estimate each of their probabilities. In **activity 2**, students further investigate the landing positions by playing a modified game of golf with the pigs and then designing their own hole for a golf course. Experimental probability is incorporated into **activity 3**, as students determine the likelihood of each landing position. Over the course of these activities, students make conjectures about situations and test them through data collection and experiments, as advocated in

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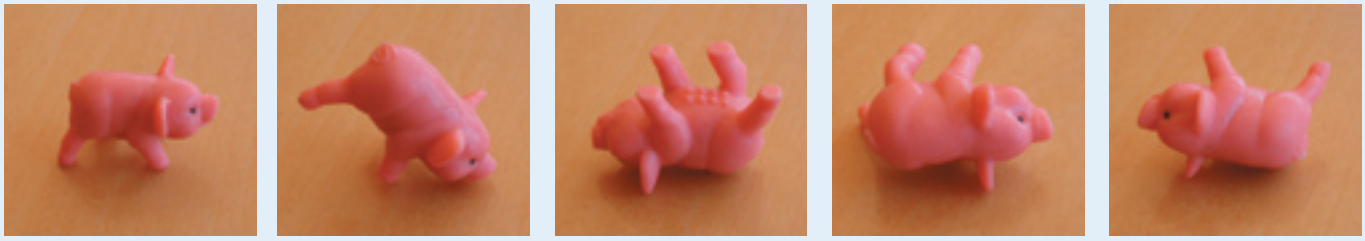
Edited by **Denisse R. Thompson**, thompson@tempest.coedu.usf.edu, Mathematics Education, University of South Florida in Tampa, and **Gwen Johnson**, gjohnson@coedu.usf.edu, Secondary Education, University of South Florida. This department provides activities for students in grades 5–9. The material may be reproduced by teachers for classroom use. Readers who have developed successful classroom activities are encouraged to submit manuscripts in the format seen here. Send submissions by accessing **mtms.msubmit.net**.

Fig. 1 The five landing positions



(a) (b) (c) (d) (e)

Principles and Standards for School Mathematics (NCTM 2000).

ACTIVITY SHEET 1: EXPLORING SAMPLE SPACE WITH TOY PIGS

To start the activity, students should consider the miniature toy pigs in **figure 1**. Ask the class, "If you were to toss this pig in the air, do you think it would land on its feet?" Students immediately begin tossing the pig, and we hear comments like the following:

- No, my pig landed on its side.
- Yes, it did land on its feet!

As students begin the activity, they should work with a partner to determine the possible landing positions. Students should also estimate the probability of each landing position with the following two probability principles in mind:

Probability Principle 1: An impossible event is an event with a probability of 0. A certain event is an event with a probability of 1. All other events have a probability between 0 and 1.

Probability Principle 2: The sum of the probabilities of all the outcomes in a sample space equals 1.

To correct misconceptions about probability, it is useful for students to make predictions and then compare their predictions with actual outcomes (NCTM 2000). In the case of the toy pigs, students might initially believe that each of the five landing positions is equally likely (see **fig. 1**). Hence, each would have a 1/5 probability of occurring. In addition, students may not recognize that they can distinguish between the two sides of the pig. As students conduct their experiments, incorrect assumptions are discovered through their data collection. For example, some students incorrectly believe that each side landing position has a probability of 1/2 and therefore their probabilities do not sum to 1. We remind them about Probability Principle 2 so that revised probabilities can be determined.

After completing **activity 1**, students should have identified the different landing positions and estimated possible probabilities (see **fig. 2**). Sharing solutions as a class leads to a discussion of common terminology when describing each of the landing positions. In addition, the meaning of the expression *equally likely outcomes* (outcomes that have the same probability of occurring) should be addressed. To investigate the probabilities of the landing positions in a unique way, students use the toy pigs to play a version of golf in **activity 2**.

ACTIVITY 2: PLAYING GOLF WITH TOY PIGS

Activity 2 begins with students playing a version of golf with a partner. A "hole" is completed when the landing positions satisfy the rule for that hole.

Fig. 2 One group's solution to activity 1 with incorrect estimated probabilities

Landing Position	Estimated Probability
Feet	1/5
Side 1	1/2
Side 2	1/2
Snout	0
Back	1/5

Each toss of the three pigs is considered a "stroke." The teacher may have to explain a few golf principles to students:

- Par is the number of strokes that a good golfer would need to put the ball in the cup.
- The player with the lowest number of total strokes is the winner.

As students simulate playing golf using toy pigs, they begin to realize that each landing position is not equally likely. For example, landing in the snout position (see **fig. 1b**) rarely occurs, making hole 7 more difficult; thus, it is a par 5.

After our students finished playing golf, they discussed whether hole 6 seemed like a par 3 hole (i.e., rolling the pigs three times is needed to satisfy the rule). Many students stated that it took more than three strokes to complete the hole, particularly because the rule "one pig lands on its feet" was difficult to

Fig. 3 One group's hole design

Rule	Par (2, 4, or 5)
At least two pigs land on their backs	*5*

Fig. 4 Combined results from two groups' trials of rolling pigs

Landing Position	Total	Experimental Probability
Snout	1	$\frac{1}{100} = .01$
Back	31	$\frac{31}{100} = .31$
Feet	8	$\frac{8}{100} = .08$
On the right	29	$\frac{29}{100} = .29$
On the left	31	$\frac{31}{100} = .31$
	100	$\frac{100}{100} = 1$

obtain. Students suggested changing "one pig lands on its feet" to "one pig lands *on its left side*." In general, students always changed "one pig lands on its feet" to a different landing position.

To explore experimental probability further, students had to design their own hole for the golf course. This activity requires students to estimate the expected number of strokes on the basis of their initial experimental probability of each landing position. While playing the game, students also should develop intuitive ideas about probabilities for the different landing positions. Students enjoyed designing a hole for the golf course, and this task provided an opportunity for them to be creative. In addition, when another group played their hole, they found out how well they determined par (see **fig. 3**). The group that designed the hole in **figure 3** found that its par was too easy to obtain, so it was lowered from 5 to 3. Other groups had to redesign their holes, often making elaborate changes by adjusting the landing positions.

ACTIVITY 3: EXPLORING EXPERIMENTAL PROBABILITIES WITH TOY PIGS

At this point, students are familiar with the toy pigs, and it is natural for them to want to know the experimental probability of each landing position. In **activity 3**, students roll two pigs twenty-five times, as they might roll dice. They combine their results with those of another group and compare their individual and combined group probabilities (see **fig. 4**). Discrepancies between students' data will lead to discussion about sample size and Bernoulli's law of large numbers (which states that as the number of experiments increases, the experimental probability approaches the theoretical probably). As our students completed this activity, they concluded that a theoretical probability for each landing position cannot be determined.

CONCLUSION

As students explore the probability of each landing position, interesting questions can be posed. For example, does it matter that a theoretical probability cannot be determined? Are there other natural phenomena for which theoretical probabilities cannot be found? Middle-grades students need to encounter situations in which the probability of each outcome is unknown. They should also develop the ability to articulate their reasoning about data. *Principles and Standards* advocates that students need to understand the thinking of their peers and relate it to their own reasoning. Playing golf with toy pigs requires students to work together to design their own golf course hole, then analyze another group's design. Through-

out these explorations with miniature toy pigs, students collect and analyze data to investigate unknown probabilities and build their probabilistic understanding.

Note: The miniature toy pigs can be found in an activity kit called Pig Pals (Whyte 2006). Included in the kit is an activity booklet for grades K–2, which contains an exercise called Piggy Golf. We found the idea of playing golf with pigs novel and decided to incorporate it into probability activities applicable to middle-grades students.

SOLUTIONS TO THE ACTIVITY SHEETS

Activity 1

1. Students should get five different landing positions. However, their labeling may vary. One way to label the way the pigs could land would be snout, feet, back, right side, and left side.
2. Answers will vary.
3. Answers will vary. Possible responses should include these: for *similar*, "You don't know what outcome will turn up"; for *different*, "Some pig positions don't happen very often. For a die, you have the same chance of rolling a 1, 2, 3, 4, 5, or 6. Each one happens about as often as any other."

Activity 2

1. Answers will vary. Students should find that the most likely landing positions will be on the right side, left side, or back.
2. Answers will vary. Many students will respond it was not a par 3, and they will change one pig landing on its feet to another landing position.
3. Answers will vary.
4. Answers will vary.

Activity 3

1. Answers will vary.
2. No. They have different probabilities.
3. See **figure 4**.
4. Answers will vary. The combined probabilities from the student groups should be fairly close to one another.
5. The experimental probability is getting closer to the "theoretical" or "true" probability.
6. No, the true theoretical probability is unknown.

REFERENCES

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics.* Reston, VA: NCTM, 2000.

Whyte, Donna. *Pig Pals*. Peterborough, NH: Crystal Springs Books, 2006.

activity sheet 1

Name _____

Exploring Sample Space with Toy Pigs

Use Probability Principles 1 and 2 as you work with a partner to answer the following questions.

> *Probability Principle 1:* An impossible event has a probability of 0. A certain event has a probability of 1. All other events have probabilities between 0 and 1.
>
> *Probability Principle 2:* The sum of the probabilities of all the outcomes in a sample space equals 1.

1. Toss one pig a number of times to determine the possible landing positions. For each landing position, estimate the probability of the pig landing in this position. Record your results in the table.

Landing Position	Estimated Probability

2. On the basis of your results, do you think each landing position is equally likely? Why or why not?

3. How is tossing a pig similar to rolling a die? How is it different?

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activity sheet 2

Name _____

Playing Golf with Toy Pigs

Play golf with a partner using the following directions. Each player needs three pigs.

To play golf with pigs, start with hole 1. Each "hole" is represented by a combination of landing positions. The player who completes the 9 holes in the fewest strokes is the winner. Take turns rolling all three pigs at once. One person should roll until he or she completes the hole, then it is the partner's turn. A hole is completed either when the pigs land in the described position for each hole or when you have rolled the pigs 15 times. Exclude each pig after it has landed in the desired position. The number of "strokes" is the number of times that the pigs have to be rolled to satisfy the rule of the hole. Record the number of strokes for each hole on the scorecard below.

Golf Scorecard

Hole Number	Rule	Par	Total Strokes
1	One pig lands on its back, and one pig lands on its right side.	3	
2	All three pigs land on their backs.	5	
3	All three pigs land in different positions.	3	
4	At least two of the pigs land on their feet.	5	
5	All three pigs land on their feet or backs. They do not all have to be in the same position (only on their feet or only on their back).	3	
6	One pig lands on its back, one pig lands on its right side, and one pig lands on its feet.	3	
7	One pig lands on its snout.	5	
8	Two pigs land in the same side position and the other one is on its back, on its feet, or on its snout.	5	
9	All three pigs land in the same side position (either on the right or on the left).	4	
	TOTAL	36	

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activity sheet 2 (continued)

After playing golf with pigs, answer the following:

1. Which landing position seems most likely? Why?

2. Did hole 6 seem to be assigned the correct par? If yes, explain why. If not, determine how you would change one of the landing positions to make it a par 3 hole.

3. Design a hole for the golf course. It should be a par 3, 4, or 5. Write the rule and the par for your new hole.

Rule	Par (3, 4 or 5)

4. Let another group play your hole.

a. If the other group's score is within two strokes of par (± 2 strokes), do not change your hole. Explain the reasoning behind your hole's design.

b. If the other group's score is not close to par, redesign your hole, and explain your changes.

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activity sheet 3

Name _____

Exploring Experimental Probabilities with Toy Pigs

Work with a partner to answer the following questions.

1. Roll two pigs 25 times for a total of 50 outcomes and record the landing position on each roll. Find the experimental probability of each landing position.

Landing Position	Tally	Total	Experimental Probability

2. Are the landing positions equally likely? Explain.

3. Combine your results with those of another group. Calculate the new experimental probability.

Landing Position	Total	Experimental Probability

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activity sheet 3 (continued)

4. How do the individual and the combined group probabilities compare?

The *Law of Large Numbers* (Bernoulli's theorem): If an experiment is repeated a large number of times, the experimental probability of a particular outcome approaches a fixed number as the number of repetitions increases.

5. What does Bernoulli's theorem mean for the probability of each landing position?

6. Is there a way to determine the exact probability of each landing position? Explain your thinking.

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