Similarity

- **Parallel Projection Theorem:**
  Let l, m, and n be distinct parallel lines. Let t be a transversal that cuts these lines at A, B, and C, respectively, and let t' be a transversal cutting these lines at A', B', and C', then

$$\frac{\overline{AB}}{\overline{AC}} = \frac{\overline{A'B'}}{\overline{A'C'}}$$

- **Side-Splitting Theorem (SST):**
  If a line parallel to a side of a triangle intersects the other two sides in distinct points, then it splits these sides into proportional segments.

- Two triangles $\triangle ABC$ and $\triangle DEF$ are **similar** if their corresponding angles are congruent and $\dfrac{\overline{AB}}{\overline{DE}} = \dfrac{\overline{BC}}{\overline{EF}} = \dfrac{\overline{CA}}{\overline{FD}} = r$.
- r is called the scale factor (similarity ratio between $\triangle ABC$ and $\triangle DEF$).
- Notation: $\triangle ABC \sim \triangle DEF$ means the triangles are similar.
- **Converse of SST:**
  If a line divides two sides of a triangle proportionally, then the line is parallel to the third side.

- **Angle-Angle (AA) Similarity Condition:**
  If two angles of one triangle are congruent to two angles of a second triangle, then the triangles are similar.
- **Side-Side-Side (SSS) Similarity Condition:**
  If corresponding sides of two triangles are proportional, then the corresponding angles are congruent and the triangles are similar.
- **Side-Angle-Side (SAS) Similarity Condition:**
  Given two triangles, if two pairs of corresponding sides are proportional and the included angles are congruent, then the triangles are similar.

- **Theorem:** If $\dfrac{a_1}{b_1} = \dfrac{a_2}{b_2} = \cdots = \dfrac{a_n}{b_n} = r$, then

$$\frac{a_1 + a_2 + \cdots + a_n}{b_1 + b_2 + \cdots + b_n} = r$$

- **Perimeter Theorem:**
  If $\triangle ABC \sim \triangle DEF$ and r is the scale factor, then

$$\frac{\text{Perimeter of } \triangle ABC}{\text{Perimeter of } \triangle DEF} = r$$

- **Area Theorem:**
  If $\triangle ABC \sim \triangle DEF$ and r is the scale factor, then

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEF} = r^2$$

- A transformation is called a **Similarity Transformation with scale factor r** (similarity ratio) if for any two points P and Q and their images P' and Q' we have that $\mathrm{distance}(P, Q) = r \cdot \mathrm{distance}(P', Q')$
- Similarity transformations preserve angle measure.
- Two geometric figures are **similar** if there exists a similarity transformation taking one figure to the other. In particular $\triangle ABC \sim \triangle DEF$ if and only if there is a similarity transformation taking one triangle onto the other.