Review for Post-Test
Year 2 (2014-2015)

Topic #1 → Systems

Multiple Choice

① $2x - 3y < 9 \rightarrow -3y < -2x + 9$ [Subtract $2x$]

$y > \frac{2}{3}x - 3$ [Divide by -3: Note inequality "flips"]

↑

y-intercept $= -3$
Slope $= \frac{2}{3}$
Shading is Above the line ($>$)
line is dotted

$2x + 3y \geq 3 \rightarrow 3y \geq -2x + 3$ [Subtract $2x$]

$y \geq -\frac{2}{3}x + 1$ [Divide by 3]

↑

y-intercept $= 1$
Slope is $-\frac{2}{3}$
Shading is above the line ($\geq$)
line is Solid

$\Rightarrow$ (D)

② $5x + 2y = 6$
$x - y = 4$ (Mult by 2) $\rightarrow$ $5x + 2y = 6$
$2x - 2y = 8$

now Add & the y's "drop out"

$7x = 14$
$x = 2$

now substitute $x = 2$ into either equation to find y.

$5x + 2y = 6$
$5(2) + 2y = 6$ → $10 + 2y = 6$
$2y = -4$
$y = -2$

$\therefore$ (D)

③

$$\begin{array}{r} 4x-3y=6 \\ x+3y=9 \\ \hline 5x=15 \\ x=3 \end{array}$$

$$\begin{array}{r} x+3y=9 \\ 3+3y=9 \\ 3y=6 \\ y=2 \end{array} \quad \Rightarrow \text{ 1 solution} \quad \begin{array}{r} x=3 \\ y=2 \end{array}$$

$\therefore$ (A) the system is consistent [it has at least 1 solution]
it is independent [it has 1 solution]

Open Ended

④

x = rate of boat in still water

y = rate of current

$D = R \cdot T$

Distance	Rate	Time	
33	$x+y$	3	downstream
33	$x-y$	11	upstream

$\Rightarrow$ $3(x+y)=33$ is our system
$11(x-y)=33$

$\Rightarrow$ $x+y=11$ [Divided by 3 ☺]
$x-y=3$ [Divided by 11]

$2x=14$ [Add : y's drop out]

$x=7$

$x+y=11$
$7+y=11$
$y=4$

$\therefore$ rate of boat in still water is 7 mph
rate of current is 4 mph

Multiple Choice

① $\dfrac{4}{x^2-4} + \dfrac{2}{x^2-6x+8}$

we begin by factoring the denominators.

$\dfrac{4}{(x+2)(x-2)} + \dfrac{2}{(x-2)(x-4)}$

Now, we build a common denominator → we'll need 3 factors : $(x+2)(x-2)(x-4)$

$\dfrac{4(x-4)}{(x+2)(x-2)(x-4)} + \dfrac{2(x+2)}{(x+2)(x-2)(x-4)}$

Adjusting the original fractions to our common denominator

$\dfrac{4(x-4)+2(x+2)}{(x+2)(x-2)(x-4)} \Rightarrow \dfrac{4x-16+2x+4}{(x+2)(x-2)(x-4)}$

$= \dfrac{6x-12}{(x+2)(x-2)(x-4)} \Rightarrow$

Factor out a 6.

$\dfrac{6(x-2)}{(x+2)(x-2)(x-4)}$

$\Rightarrow \dfrac{6}{(x+2)(x-4)}$ ∴ (A)

② $\dfrac{x^2+14x+48}{8x+64} \cdot \dfrac{x^2+5x+4}{x^2+10x+24}$ First, we factor.

$\dfrac{(x+6)(x+8)}{8(x+8)} \cdot \dfrac{(x+4)(x+1)}{(x+6)(x+4)}$ Now, we cancel any common factor found on top & bottom.

$\Rightarrow \dfrac{x+1}{8}$ $\therefore$ (B)

③ $\dfrac{x+3}{2x+4} - \dfrac{1}{2} = \dfrac{x-5}{x^2+2x}$ First, we factor our denominators.

$\dfrac{x+3}{2(x+2)} - \dfrac{1}{2} = \dfrac{x-5}{x(x+2)}$ So, our LCD $= 2(x)(x+2)$

Now, we Multiply Both sides by our LCD [our goal is to eliminate the fractions]

$2(x)(x+2)\left[\dfrac{x+3}{2(x+2)} - \dfrac{1}{2}\right] = 2(x)(x+2)\left[\dfrac{x-5}{x(x+2)}\right]$

this is a distributive situation ☺

$x(x+3) - x(x+2) = 2(x-5)$

$x^2 + 3x - x^2 - 2x = 2x - 10$

$x = 2x - 10$

$10 = x$ $\therefore$ (D)

④ $\dfrac{x^2-5x+8}{x-2} \Rightarrow$

$$x-2 \;\overline{)\; x^2-5x+8}$$

$$\begin{array}{r} x-3 \\ x-2 \;\overline{)\; x^2-5x+8} \\ \text{SUBTRACT} \rightarrow \quad x^2-2x \quad \downarrow \text{ Bring down} \\ \ddot{\smile} \qquad\qquad -3x+8 \\ \qquad\qquad -3x+6 \\ \hline \qquad\qquad\quad 2 \end{array}$$

$\therefore \quad x-3+\dfrac{2}{x-2} \Rightarrow$ (C)

⑤ We'll do this by synthetic division!

given k value → -1

	10	20	12	7
	↓	-10	-10	-2
	10	10	2	5
				r

← coefficients for P(x)

① copy the 10 down:

② Multiply by -1

③ write the result & add.

Repeat to the end.

Q(x) — These get us Q(x) and r

[1 power smaller than P(x) so x^2 ☺]

$10x^2+10x+2 = Q(x)$

$5 = r$

So... (A)

⑥ $\dfrac{5 + \frac{10}{x}}{\frac{x}{2} + 1}$ multiply top & bottom by LCD $[2x]$

$$\frac{2x\left[5 + \frac{10}{x}\right]}{2x\left[\frac{x}{2} + 1\right]} = \frac{10x + 20}{x^2 + 2x} \rightarrow \frac{10(x+2)}{x(x+2)} = \frac{10}{x}$$

now factor

$\therefore$ (D)

⑦ Back to synthetic division.

missing x^2 term ☺

-2 | dividing by $x + 2$

note: the $-$ sign.

4	2	0	−4	−18
	−8	12	−24	56
4	−6	12	−28	38

↓ used to write quotient again 1 power smaller; so x^3

the remainder

$\therefore$ Quotient $= 4x^3 - 6x^2 + 12x - 28$

Remainder $= 38$

Topic #3 → Vectors And Matrices

Multiple Choice

① $\begin{bmatrix} -2 & -6 \\ 5 & 5 \end{bmatrix} \cdot \begin{bmatrix} 3 & 3 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} (-2)(3)+(-6)(0) & (-2)(3)+(-6)(2) \\ (5)(3)+(5)(0) & (5)(3)+(5)(2) \end{bmatrix}$

→ work across as you ↓ work down

$= \begin{bmatrix} -6+0 & -6+(-12) \\ 15+0 & 15+10 \end{bmatrix}$

$= \begin{bmatrix} -6 & -18 \\ 15 & 25 \end{bmatrix}$ ∴ (A)

② Dimensions = Rows by Columns

3 5

rows go across
columns go down

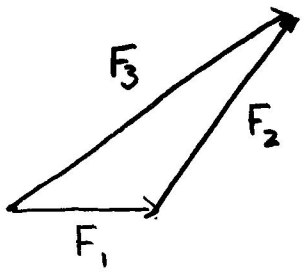
∴ 3×5 ⇒ (C)

③ (3, 2)

↓ row 3 ↘ column 2

$\Rightarrow \begin{bmatrix} 1 & 2 & 4 & 6 & 2 \\ 13 & 1 & 8 & 5 & -6 \\ -1 & 1 & 5 & 3 & 10 \end{bmatrix} \Rightarrow 1$ ∴ (D)

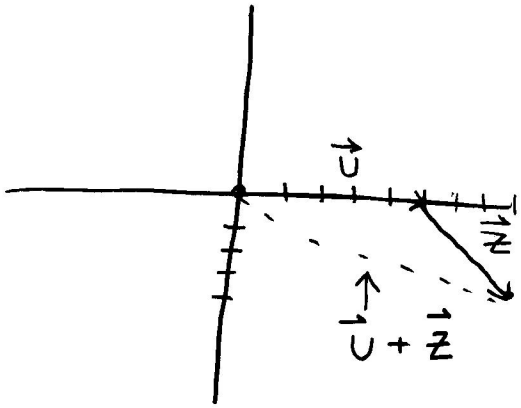
④



$\Rightarrow F_1 + F_2 = F_3$ ∴ (B)

⑤ $AB = \begin{bmatrix} \text{Total cost} & M \\ " & T \\ " & W \\ " & R \\ " & F \end{bmatrix} \Rightarrow$ (A)

⑥ $\vec{U} + \vec{Z} \Rightarrow$



$\Rightarrow$ right 8 down 4

$\Rightarrow$ (B)

Open Ended

⑦ A] $B \cdot Y = \begin{bmatrix} 3 & -2 & 4 \\ -1 & -1 & 0 \\ 2 & 3 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} (3)(1)+(-2)(2)+(4)(3) \\ (-1)(1)+(-1)(2)+(0)(3) \\ (2)(1)+(3)(2)+(5)(3) \end{bmatrix}$

→ work across ↓ work down

$= \begin{bmatrix} 3-4+12 \\ -1-2+0 \\ 2+6+15 \end{bmatrix}$

$= \begin{bmatrix} 11 \\ -3 \\ 23 \end{bmatrix}$

⑦ continued.

B) $A + B = \begin{bmatrix} 1 & -1 & 4 \\ -1 & 2 & 3 \\ 4 & 3 & -2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & 4 \\ -1 & -1 & 0 \\ 2 & 3 & 5 \end{bmatrix}$

$= \begin{bmatrix} 1+3 & -1+(-2) & 4+4 \\ -1+(-1) & 2+(-1) & 3+0 \\ 4+2 & 3+3 & -2+5 \end{bmatrix}$

$= \begin{bmatrix} 4 & -3 & 8 \\ -2 & 1 & 3 \\ 6 & 6 & 3 \end{bmatrix}$

C) $3A - 2B = 3\begin{bmatrix} 1 & -1 & 4 \\ -1 & 2 & 3 \\ 4 & 3 & -2 \end{bmatrix} - 2\begin{bmatrix} 3 & -2 & 4 \\ -1 & -1 & 0 \\ 2 & 3 & 5 \end{bmatrix}$

$= \begin{bmatrix} 3 & -3 & 12 \\ -3 & 6 & 9 \\ 12 & 9 & -6 \end{bmatrix} - \begin{bmatrix} 6 & -4 & 8 \\ -2 & -2 & 0 \\ 4 & 6 & 10 \end{bmatrix}$

$= \begin{bmatrix} 3-6 & -3-(-4) & 12-8 \\ -3-(-2) & 6-(-2) & 9-0 \\ 12-4 & 9-6 & -6-10 \end{bmatrix} = \begin{bmatrix} -3 & 1 & 4 \\ -1 & 8 & 9 \\ 8 & 3 & -16 \end{bmatrix}$

7 (Concluded)

D] $X \cdot Y = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $\Rightarrow$ Not possible

$3 \times 1 \quad\quad 3 \times 1$

these numbers must be = in order to multiply matrices ☺

Topic #4 → Transformations

Multiple Choice

① "slides" are translations So... (A)

② reflecting an ordered pair over the y-axis results in:

1) no change to the y-value
2) the x-value is "negated" – the sign changes.

So P: $(-8,-4) \rightarrow P'(8,-4)$
Q: $(1,-6) \rightarrow Q'(-1,-6)$
R: $(7,-2) \rightarrow R'(-7,-2)$

$\therefore$ (D)

③ K has coordinates (2,3)
translating 4 units left and 3 units down $\Rightarrow (2-4, 3-3)$

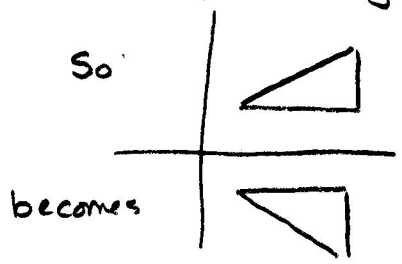
or

$(-2,0)$

$\therefore$ (A)

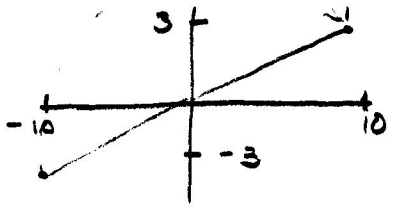
⑤ Reflecting with respect to the x-axis results in:

1) no change in the x-value
2) the y-value is "negated" - the sign changes



∴ (D)

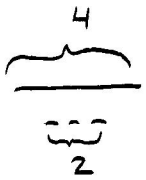
⑥



In effect a 180° rotation about the origin causes a sign change on both x and y

⇒ (10, 3) ∴ (B)

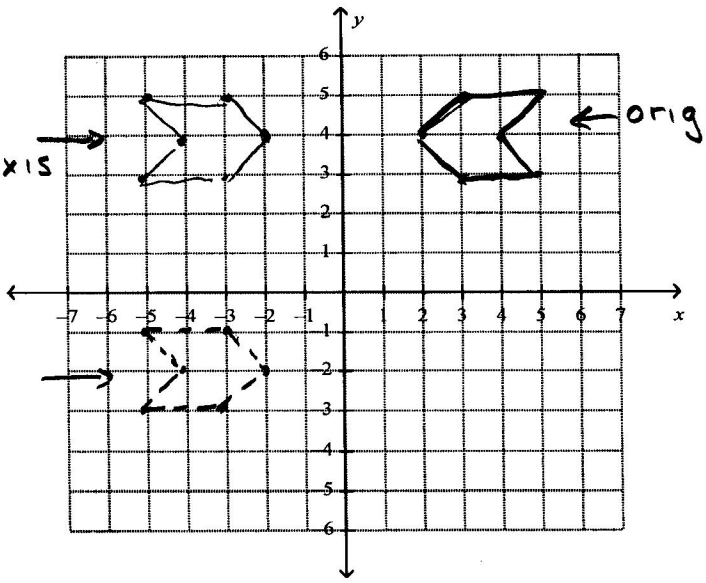
⑦ Dilation → let's compare some "side lengths"



$\frac{2}{4}$ or $\frac{1}{2}$ ∴ (B)

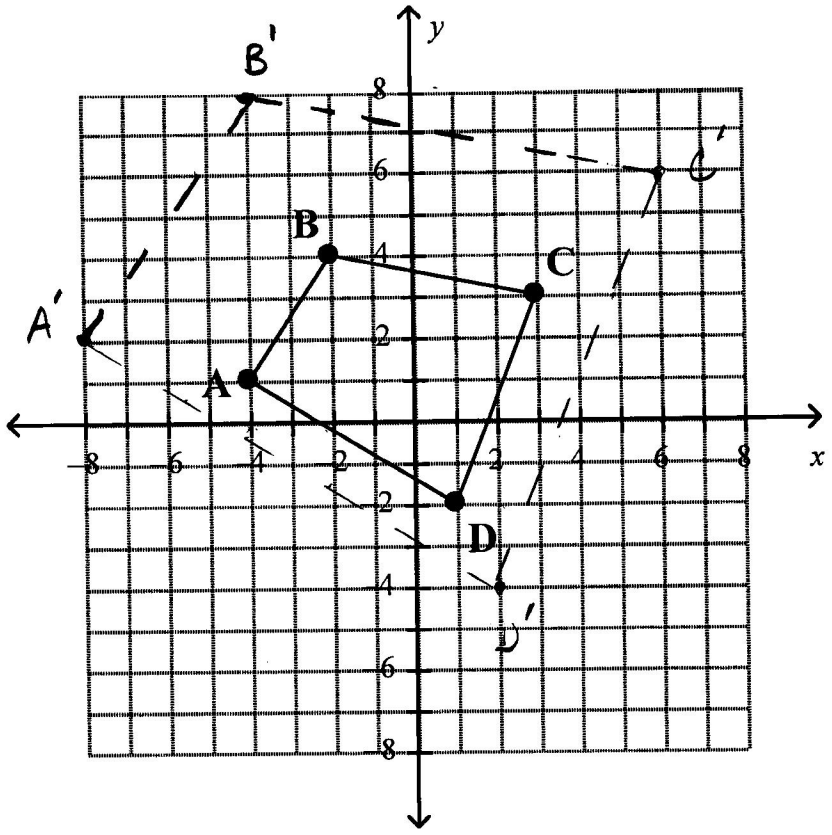
Open Ended

⑧ A-D [on graph]



E) (-2, -2)
(-3, -1)
(-5, -1)
(-4, -2)
(-5, -3)
(-3, -3)

⑨ Scale factor of 2 → "double each x-value, double each y-value."



A' → (−8, 2)

B' → (−4, 8)

C' → (6, 6)

D' → (2, −4)

⑩ W' → (−2 + 6, 4 − 3) ⇒ (4, 1) = W'

X' → (−6 + 6, 7 − 3) ⇒ (0, 4) = X'

Y' → (1 + 6, 2 − 3) ⇒ (7, −1) = Y'

Z' → (4 + 6, −5 − 3) ⇒ (10, −8) = Z'

Topic # 5 → Statistics (sampling types; correlation vs. causation)

Multiple Choice

① the population is the large group whose opinion you seek.

∴ (A)

② the sample is the "subset" of the population from which you gather data

∴ (B)

③ the statistic is the data result compiled from the sample

∴ (C)

④ the parameter refers to the characteristic about the population that we seek to examine.

∴ (D)

⑤ this is a voluntary response sample - each person who contributes does so voluntarily.

∴ (D)

⑥ this is an uncontrolled experiment → there is no effort made to control other variables, such as overall health, etc

∴ (D)

⑦ Again, since other variables aren't controlled, it isn't possible to conclude a cause & effect relationship between taking Tamiflu and reduced flu symptoms.

∴ (B)

Open Ended

⑧ A] put each number on a card; put all 500 cards in a hat; select 20 cards from the hat.

⑧ Continued.

B) Put the 500 #s into 5 clusters: 1-100, 101-200, 201-300, 301-400, 401-500. Select 4 cards from each cluster → yields 20.

c) randomly select a number from 1-5, Then randomly select 20 cards from that cluster.

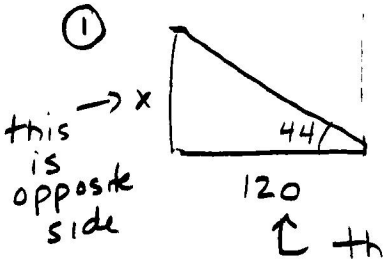
1 → cluster 1-100
2 → " 101-200
3 → " 201-300
4 → " 301-400
5 → " 401-500

D) Select a starting spot (say # 33) then select every 4th no until you have a sample of 20 numbers.

⑨ To conclude causation; a control group must be used. participants in the study must be randomly placed [$\frac{1}{2}$ in control group, $\frac{1}{2}$ in experimental group]. Efforts must be made to control conflicting variables - so as to allow the researcher to conclude that the experimental treatment is "causing" a change in outcome.

A researcher may determine an association by doing an observational study that might not even involve an experiment or a control group. Statistically speaking many things are associated or correlated, but a rigid experiment must be constructed & carried out if one seeks to claim causation.

Multiple Choice

① 

We will work from the 44° angle.
We know the adjacent side = 120
and seek the opposite side

T O A / so we use tan

remember
S O H / C A H / T O A

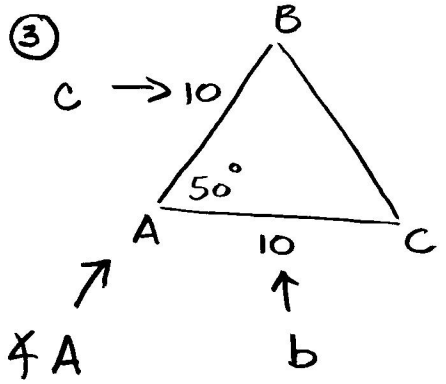
$$\tan 44 = \frac{x}{120}$$

$$120 \cdot \tan 44 = x$$ [Now, it's calculator time]

$$115.88 = x$$

∴ (D)

② SIN = opposite over hypotenuse ∴ (D)

③ 

→ $\text{AREA} = \frac{1}{2} bc \sin A$

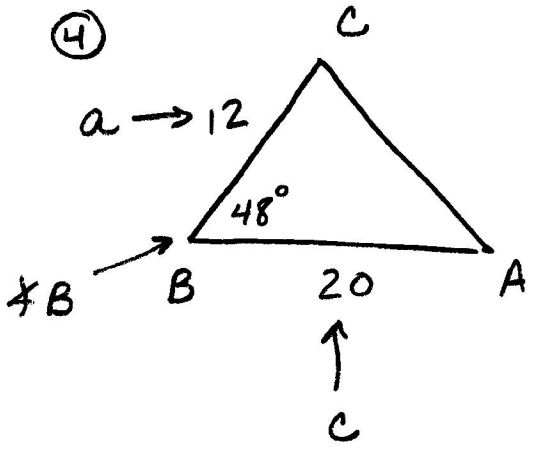
Just substitute in values and crank up the calculator

$$A = \frac{1}{2}(10)(10)(\sin 50)$$

$$= 38.30 \text{ m}^2$$

∴ (C)

④



→ $b^2 = a^2 + c^2 - 2ac\cos B$

Again, just substitute in the values for a, c, and ∡B and grab your calculator.

$b^2 = (12)^2 + (20)^2 - (2)(12)(20)(\cos 48)$

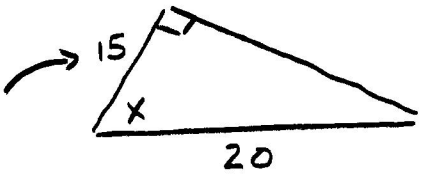
$b^2 = 222.817\ldots$

taking the square root →

$b = 14.927$

$\therefore$ (B)

⑤



this is the adjacent side

this is the hypotenuse

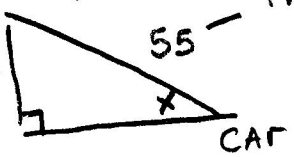
So we use C A H

↑ cosine

$\cos X = \frac{15}{20}$ So we type $\cos^{-1}\left(\frac{15}{20}\right)$ on our calculator to find ∡X

$X = 41.409$ $\therefore$ (A)

⑥



this is the hypotenuse

this is the opposite side

So we use S O H

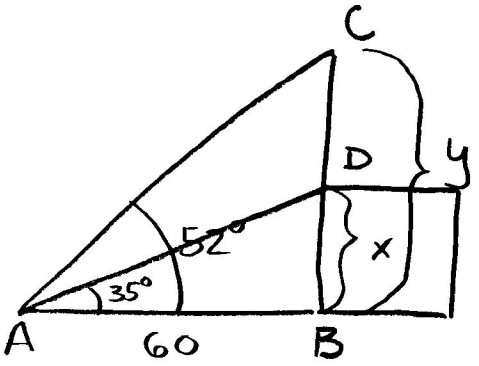
↑ sine

$\sin X = \frac{30}{55}$ as above we type $\sin^{-1}\left(\frac{30}{55}\right)$ to find x

$X = 33.06$ $\therefore$ (B)

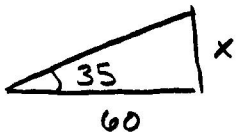
Open Ended

⑦



We'll use 2 triangles and find 2 distances - which we'll use find the length of the pole → CD.

In the little Δ



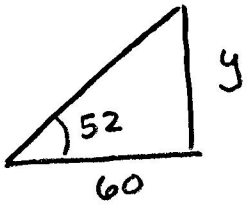
we can use Tangent to find x → the height of the building

$$\tan 35 = \frac{x}{60}$$

$$x = 60 \cdot \tan 35$$

$$x = 42.01$$

In the BIG Δ



We can use Tangent to find y → the distance from the ground to the top of the pole.

$$\tan 52 = \frac{y}{60}$$

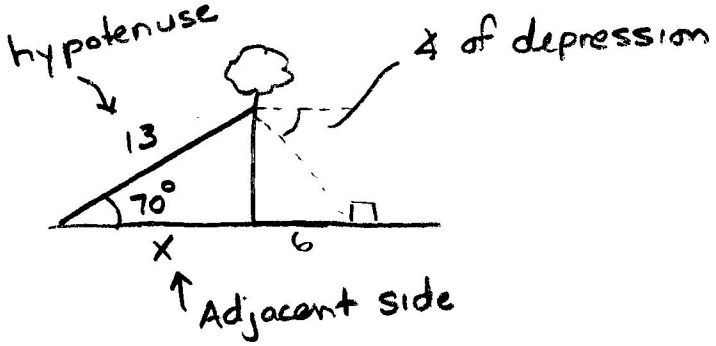
$$y = 60 \cdot \tan 52$$

$$y = 76.80$$

⇒ the height of the pole $[y - x] = 76.80 - 42.01$

$= 34.79$ m

⑧ First a sketch.



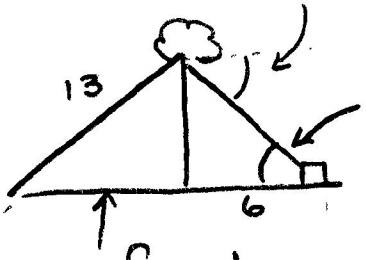
A) we seek x → the distance of the base of the ladder from the tree

$$\cos 70 = \frac{x}{13}$$

$$x = 13 \cdot \cos 70$$

$$x = 10.26 \rightarrow 10.3 \text{ m}$$

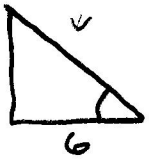
⑧ continued

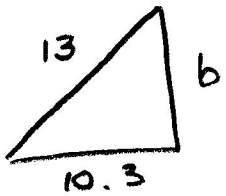


= to this ∠ [Alternate interior ∠s]

we found this to be 10.3m in part A

first we'll find this side's length →





→ let's use Pythagorean Theorem

$a^2 + b^2 = c^2$

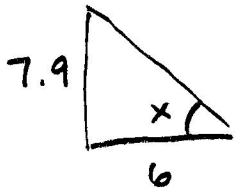
$(10.3)^2 + b^2 = 13^2$

$b^2 = 13^2 - (10.3)^2$

$b^2 = 62.91$ now take $\sqrt{}$

$b = 7.93$ or 7.9

so we have



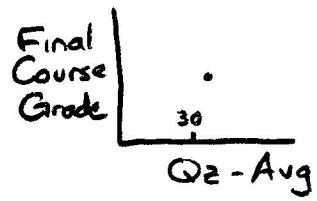
$\rightarrow \tan x = \frac{7.9}{6}$

$\Rightarrow x = \tan^{-1}\left(\frac{7.9}{6}\right)$

$x = 52.78 \Rightarrow 52.8^\circ$

Topic #7 → Categorical Variables and Quantitative Variables.

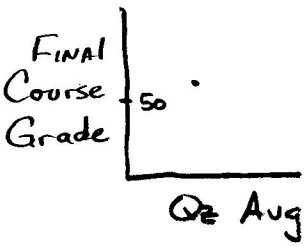
Multiple Choice

① 

→ we seek lowest quiz Avg [i.e the dot with Smallest x-value

It looks to be near 30.

so (A) between 30 & 40.

② 

→ we seek the lowest final course grade [i.e the dot with the lowest y-value.

It looks to be near 50

so (B) between 50 & 60.

③ the regression equation is $y = 0.622x + 35.69 \rightarrow y = mx + \underline{b}$

the y-intercept is $\underline{b}$

so when x (quiz average) = 0, the predicted y (final course grade) is 35.69

$\therefore$ (A)

④ the regression equation is

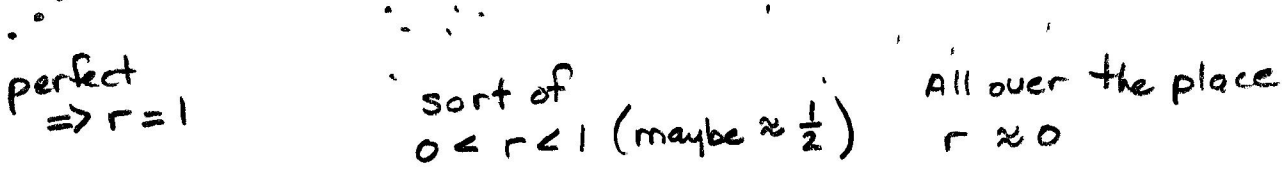
$$y = \underline{0.622}x + 35.69 \rightarrow y = \underline{m}x + b$$

the slope (m) = .622. remember slope = $\frac{\text{change in } y}{\text{change in } x}$.

so we write .622 as $\frac{.622}{1}$ ← change in y (final course Avg.) / ← change in x (quiz average)

$\therefore$ for each 1% change in quiz average, we predict final course average to increase by .622% → (C)

⑤ r is correlation value and describes how well the data fits a straight line.



perfect
$\Rightarrow r = 1$

sort of
$0 < r < 1$ (maybe $\approx \frac{1}{2}$)

All over the place
$r \approx 0$

Looking at the scatter plot, we see a decent linear trend

$\therefore \quad \frac{1}{2} \leq r \leq 1$ so (D)

Combination of Multiple Choice & Open ended

⑥

	8:00 Class	9:00 Class	TOTAL
Earned an A	18	12	30
Did not earn an A	4	6	10
TOTAL	22	18	40

Just Add :)

⑦ overall $\frac{30}{40}$ earned A's so (75%)

⑧ $a = \frac{18}{22} \approx 81.8\%$; $b = \frac{12}{18} = 66.6\%$; $c = \frac{30}{40} = 75\%$

So... $a > c > b$ or equivalently $b < c < a$ $\therefore$ (C)

⑨ $x = \frac{18}{30} = 60\%$; $y = \frac{4}{10} = 40\%$; $z = \frac{22}{40} = 55\%$

So... $x > z > y$ or equivalently $y < z < x$ $\therefore$ (C)

Open Ended

(10) (A) Just Add! ☺

	Type A	Type B	Type AB	Type O	TOTAL
Junior	5	6	3	11	25
Senior	2	4	1	8	15
TOTAL	7	10	4	19	40

(B) % of donors who are juniors.

$\frac{25}{40} = 62.5\%$

(C) % of donors who had type AB blood.

$\frac{4}{40} = 10\%$

(D) Which class had a higher % of type O blood?

Jrs $\frac{11}{25} = 44\%$ Srs $\frac{8}{15} = 53.\overline{3}\%$

so, a larger percentage of Seniors were type O blood.

(11) A) Our regression equation is:

$y = 0.612x + 31.53$

y ↑ Final Course Grade; x ↑ Quiz Average

we just "plug-in" 80 as x, and solve for y

$y = 0.612(80) + 31.53 \Rightarrow$ $y = 80.49$ or maybe 80

(11B) $y = .612x + 31.53$ → This time we're given a value of y (95) and asked to find x.

"plug-in" 95 for y and do some algebra to solve for x.
- subtract 31.53
- divide by .612

$95 = .612x + 31.53$

$63.47 = .612x$

$103.7 = x$
or
maybe 104

(11C) $y = .612x + \underline{31.53} \Rightarrow y = mx + b$ ← the y-intercept.

so we'd say that a student who has a quiz average ("x-value") of 0, would have a predicted final course grade ("y-value") of 31.53.

(11D) $y = \underline{.612}x + 31.53 \Rightarrow y = \underline{m}x + b$ ↳ the slope

recall that slope = $\frac{\text{change in } y}{\text{change in } x}$ → so we have

$m = .612 = \frac{.612}{1}$

so, we'd say that an increase of 1% in quiz average (x) would lead to an increase of .612% in final course grade (y).